Influence of the magnetopolaron effect on light reflection and absorption by a wide semiconductor quantum well

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Light reflection and absorption spectra by a semiconductor quantum well (QW) , which width is comparable to a light wave length of stimulating radiation, are calculated. A resonance with two close located exited levels is considered. These levels can arise due to splitting of an energy level of an electron-hole pair (EHP) due to magnetopolaron effect, if the QW is in a quantizing magnetic field directed perpendicularly to the QW plane. It is shown that unlike a case of narrow QWs light reflection and absorption depend on a QW width d. The theory is applicable at any ratio of radiative and non-radiative broadenings of electronic excitations.

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I. INTRODUCTION

At light transmission through a QW some characteristics occur in reflected and transmitted light, on which it is possible to judge about electronic processes proceeding in a QW [1-4]. Most interesting results turn out when energy levels of electronic system are discrete. It takes place in a quantizing magnetic field directed perpendicularly to a QW plane or at taking into account excitonic levels in a zero magnetic field.

Two close located energy levels arise in a case of a magnetophonon resonance [5], when

$$\omega_{LO} = j\,\Omega, \tag{1}$$

where ω_{LO} is the longitudinal optical (LO) phonon frequency, j is the integer and

$$\Omega = |e|H/cm_{e(h)} \tag{2}$$

is the cyclotron frequency, e is the electron charge, $m_{e(h)}$ is the electron (hole) effective mass.

The modern semiconductor technologies allow to make QWs of high quality, when radiative broadening of an absorption line may be comparable to contributions of non-radiative mechanisms or to exceed them. In such situation it is impossible to be limited by linear interaction of an electron with electromagnetic field and it is necessary to take into account all the orders of this interaction [6-22].

Light pulses and monochromatic radiation were considered as stimulating light. One, two and large number of exited levels were taken into account. Results of all previous works, except [19,20], are fair for rather narrow QWs, when the condition

$$\kappa d << 1, \tag{3}$$

is satisfied (κ is the module of a light wave vector, d is the QW width). As calculations show, in case of narrow QWs only positions of reflection and absorption peaks depend on the QW width, but not their height and form.

In [19,20] the theory of light reflection and absorption is constructed for wider QWs for which

$$\kappa d \geq 1. \tag{4}$$

In both works the interaction of light with one exited level was considered, in [19] - at a monochromatic irradiation, in [20] - at a pulse irradiation. Under condition (4) results begin to depend on the QW width d.

In the present work we investigate theoretically reflection, absorption and transmission of monochromatic light through a wide QW in a case, when the interaction of light with two close located energy levels is essential. We compare new results to conclusions of [21] devoted to research of similar problems in narrow QWs.

II. STATEMENT OF A TASK AND INITIAL EQUATIONS

The case of normal incidence of light on a semiconductor QW surface, located in a plane xy, is considered. QW may be in a zero or quantizing magnetic field perpendicular to a QW surface. Temperature is close to 0. Light excites electron-hole pairs (EHPs). In the theory interband matrix elements $\mathbf{p}_{cv}$, describing an electron transition from valence band in a conductivity band (i.e. the EHP creation) are essential. As well as in previous works, the following model will be used. Vectors $\mathbf{p}_{cv}$ for two degenerated valence bands $v = I$ and $v = II$ are

$$\mathbf{p}_{cvI} = \frac{p_{cv}}{\sqrt{2}}(\mathbf{e}_x - i\mathbf{e}_y),$$
$$\mathbf{p}_{cvII} = \frac{p_{cv}}{\sqrt{2}}(\mathbf{e}_x + i\mathbf{e}_y), \tag{5}$$

where $\mathbf{e}_x$ and $\mathbf{e}_y$ are the unite vectors along x and y axis, p_{cv} is the real value. This model corresponds to heavy

holes in a semiconductor with the zinc blend structure, if the z axis is directed along a 4-th order axis [23,24]. If to use circular polarization vectors of a stimulating light

$$\mathbf{e}_\ell = \frac{1}{\sqrt{2}}(\mathbf{e}_x \pm i\mathbf{e}_y), \tag{6}$$

a property of preservation of a polarization vector

$$\sum_{v=I,II} \mathbf{p}^*_{cv}(\mathbf{e}_\ell \mathbf{p}_{cv}) = \sum_{v=I,II} \mathbf{p}_{cv}(\mathbf{e}_\ell \mathbf{p}^*_{cv}) = \mathbf{e}_\ell p^2_{cv} \tag{7}$$

is preserved. Thus, neither EHP wave functions, nor energy levels depend on band numbers I or II.

A class of a wave function $F_\rho(\mathbf{r})$ at $\mathbf{r}_e = \mathbf{r}_h = \mathbf{r}$ is essential in the theory in the effective mass approximation ($\mathbf{r}_e(\mathbf{r}_h)$ is the electron (hole) radius - vector) [22]. We suppose that it is possible to write down

$$F_\rho(\mathbf{r}) = Q_\pi(\mathbf{r}_\perp)\phi_\chi(z). \tag{8}$$

The separation of variables is possible, if the Coulomb interaction poorly influences on the movement of these particles in xy plane. It occurs, if the quantizing magnetic field is directed along z axis and the condition

$$a^2_{exs} \gg a^2_H, \tag{8a}$$

is carried out, where

$$a_{exs} = \hbar^2 \varepsilon_0/\mu e^2$$

is the radius of a Wannier-Mott exciton in absence of a magnetic field, ε_0 is the static permittivity, μ is the reduced effective mass, $a_H = \sqrt{c\hbar/|e|H}$ is the magnetic length.

For $GaAs$ it is obtained (the parameters from [26] are used):

$$a_{exs} = 146\mathring{A}, \quad a^{res}_H = 57.2\mathring{A}, \tag{8b}$$

where a^{res}_H corresponds to magnetic field H_{res}, which is obtained from (1) at $j = 1$ for a magnetopolaron, containing an electron. According to (8b) we obtain

$$(a^{res}_H/a_{res})^2 \cong 0.154,$$

i.e. the condition (8) is carried out. Influence of the Coulomb interaction on the movement of particles in xy plane was considered in [27].

In case of free EHPs (without an account of Coulomb forces)

$$\phi_\chi(z) = \varphi^e_{\ell_e}(z)\varphi^h_{\ell_h}(z), \tag{9}$$

where $\varphi^{e(h)}_\ell(z)$ is the electron (hole) wave function corresponding to a size quantization quantum number l. For QWs of finite depth functions $\varphi^{e(h)}_\ell(z)$ are given, for example, in [25]. For infinitely deep QW (when there is no any tunnel penetration of electrons and holes in a barrier)

$$\varphi^e_{\ell_e}(z) = \varphi^h_{\ell_h}(z) = \varphi_\ell(z), \qquad \ell = 1, 2, ...$$

$$\varphi_\ell(z) = \begin{cases} \sqrt{\frac{2}{d}} \quad sin(\frac{\pi\ell z}{d} + \frac{\pi\ell}{2}), & -\frac{d}{2} \le z \le \frac{d}{2}, \\ 0 & z \le -\frac{d}{2},\, z \ge \frac{d}{2}. \end{cases} \tag{10}$$

In order to neglect by Coulomb forces at the description of particles movements along z axis, the performance of the condition

$$a_{exs} > d \tag{10a}$$

is required. Whether the conditions (4) and (10) are compatible? For example, for $GaAs$ the energy gap $\hbar\omega_g \cong 1.85eV$, the frequency of stimulating light owes to exceed this value. The module $\kappa_g = \omega_g \nu/c$ of a light wave vector corresponds to the frequency ω_g, where ν is the refraction index, c is the light velocity in vacuum. For $GaAs$ we have $\kappa_g = 3.16.10^5 cm^{-1}$, i. e. if $d = a_{exs}$, then $\kappa_g a_{exs} = 0.46$, what is comparable to unit. Thus, for $GaAs$ it is possible to neglect by Coulomb forces at movement along z axis to combine with condition $\kappa d \ge 1$ for " wide QWs " only with some stretch. But even if there are essential deviations from (9) we obtain the same qualitative results for frequency dependence of light absorption and reflection using unknown function $\phi_\chi(z)$ (see below).

We write down the stimulating electrical field extending along z axis as

$$\mathbf{E}_0(z,t) = \frac{\mathbf{e}_\ell}{2\pi}\int_{-\infty}^{\infty} d\omega e^{-i\omega t}\mathcal{E}_0(z,\omega) + c.c., \tag{11}$$

where ν is the refraction index (identical inside and outside of a QW),

$$\mathcal{E}_0(z,\omega) = 2\pi\, E_0 e^{i\kappa z}\mathcal{D}_0(\omega), \quad \kappa = \omega\nu/c. \tag{12}$$

$\mathcal{D}_0(\omega)$ may correspond to a light pulse of any form [22] and at excitation by monochromatic light with frequency ω_ℓ looks like

$$\mathcal{D}_0(\omega) = \delta(\omega - \omega_\ell). \tag{13}$$

Let us spread out a true field $\mathbf{E}(z,t)$ in the Fourier integral

$$\mathbf{E}(z,t) = \frac{\mathbf{e}_\ell}{2\pi}\int_{-\infty}^{\infty} d\omega e^{-i\omega t}\mathcal{E}(z,\omega) + c.c.. \tag{14}$$

In [22] the equation (see also [19])

$$\begin{aligned} \mathcal{E}\ \ (z,\omega) &= -\frac{i}{2}\sum_\rho \gamma_{r\pi}\int_{-d/2}^{d/2} dz'\phi_\chi(z')\mathcal{E}(z',\omega) \\ &\times\ \left\{ e^{i\kappa z}\int_{-d/2}^{z} dz'' e^{-i\kappa z''}\phi_\chi(z'') \right. \\ &+\ \left. e^{-i\kappa z}\int_{z}^{d/2} dz'' e^{i\kappa z''}\phi_\chi(z'') \right\} \\ &\times\ \{(\omega - \omega_\rho + i\gamma_\rho/2)^{-1} + (\omega + \omega_\rho + i\gamma_\rho/2)^{-1}\} \\ &+\ \mathcal{E}_0(z,\omega), \end{aligned} \tag{15}$$

for the Fourier-component $\mathcal{E}(z,\omega)$ is obtained, where $\kappa = \omega\nu/c$, the sum on indexes ρ means summation on excitation levels in a QW. A set of indexes ρ is separable on two groups

$$\rho \to \pi, \chi, \tag{16}$$

concerning, according to (8), to cross and longitudinal wave functions. In case of free EHPs the set χ contains two indexes: ℓ_e and ℓ_h.

In a quantizing magnetic field (far from magnetophonon resonance) the set π includes only one index

$$n_e = n_h = n, \tag{17}$$

where $n_e(n_h)$ is the Landau quantum number concerning to electron (hole). Close to the magnetophonon resonance a level with an index n splits in two levels with indexes $p = a$ and $p = b$ [25,21].

The equality (17) is caused by the law of preservation of quantum number n at EHP creation in case of normal light incidence on QW surface.

Far away from the magnetophonon resonance energy levels in a quantizing magnetic field are

$$\omega_\rho = \omega_g + \varepsilon_\chi/\hbar + \Omega_{\mu H}(n + 1/2), \tag{18}$$

$$\Omega_{\mu H} = \frac{|e|H}{\mu c}, \tag{19}$$

H is the magnetic field, $\mu = m_e m_h/(m_e + m_h)$. In case of free EHPs

$$\varepsilon_\chi = \varepsilon^e_{\ell_e} + \varepsilon^h_{\ell_h}, \tag{20}$$

where $\varepsilon^e_{\ell_e}(\varepsilon^h_{\ell_h})$ is the size quantized electron (hole) energy level and in approximation of infinitely deep QW (which is used at obtaining of (15))

$$\varepsilon^{e(h)}_\ell = \frac{\hbar^2\pi^2\ell^2}{2m_{e(h)}d^2}. \tag{21}$$

Close to the magnetophonon resonance for a polaron A (Fig.1), according, for example, to [21], we have

$$\omega_\rho = \omega_g + \varepsilon_\chi/\hbar + (3/2)\Omega_h + E_p/\hbar, \tag{22}$$

$$E_p = \hbar\Omega_e + \hbar\omega_{LO}/2 \pm \sqrt{(\lambda/2)^2 + A^2}, \tag{23}$$

$$\lambda = \hbar(\Omega_e - \omega_{LO}), \quad A = \Delta E/2,$$

ΔE is the polaron splitting in the exact resonance, when $\lambda = 0$. The top sign in (23) corresponds to the top polaron level $p = a$, bottom - to the bottom polaron level $p = b$.

The values $\gamma_{r\pi}$ represent some factors which are included in expressions for radiative broadenings of electronic excitations (see (24)). In [21] these broadenings

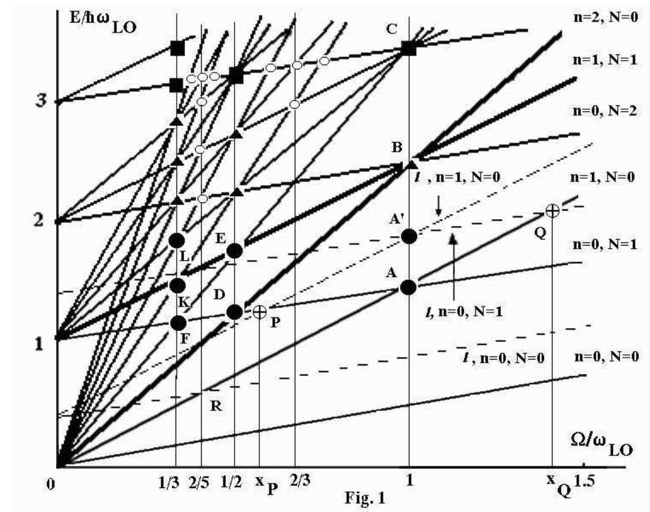

FIG. 1: Energy levels of of electron (hole)-phonon system as functions of a magnetic field. Polaron states correspond to crossing points of lines. Black circles are the twofold polarons, triangles are threefold polarons, squares are fourfold polarons. Empty circles are weak polarons. Ω is the cyclotron frequency, ω_{LO} is the LO-phonon frequency, E is the energy counted from the size quantized energy ε_ℓ. n is the Landau quantum number, N is the number of LO - phonons, ℓ, ℓ' are the size quantization quantum numbers.

$\tilde{\gamma}_{r\rho}$ are calculated in the distance and near the magnetophonon resonance. At $\mathcal{K}_\perp = 0$ (where $\mathcal{K}_\perp$ is the cross component of a quasi-momentum of an electronic excitation) we have

$$\tilde{\gamma}_{r\rho} = \gamma_{r\pi}|R_\chi(\omega_\rho\nu/c)|^2, \tag{24}$$

where

$$R_\chi(\kappa) = \int_{-d/2}^{d/2} dz e^{-i\kappa z}\phi_\chi(z). \tag{25}$$

Far from the magnetophonon resonance

$$\gamma_{r\pi} = 2\frac{e^2}{\hbar c\nu}\frac{p^2_{cv}}{m_0}\frac{\Omega_0}{\hbar\omega_g}, \quad \Omega_0 = \frac{|e|H}{m_0 c}, \tag{26}$$

i.e. it does not depend on an index $\pi = n$. Near the resonance for an excitation, consisting of the polaron A (see Fig.1) and a hole with $n = 1$

$$\gamma_{rp} = 2\frac{e^2}{\hbar c\nu}\frac{p^2_{cv}}{m_0}\frac{\Omega_0}{\hbar\omega_g}Q_{0p}, \tag{27}$$

where

$$Q_{0p} = \frac{1}{2}\left(1 \pm \frac{\lambda}{\sqrt{\lambda^2 + 4A^2}}\right), \tag{28}$$

and the top sign corresponds to the term $p = a$, bottom - to the term $p = b$. Precisely in the resonance $\lambda = 0$, and Q_{0p} is identical for terms $p = a$ and $p = b$ and is equal

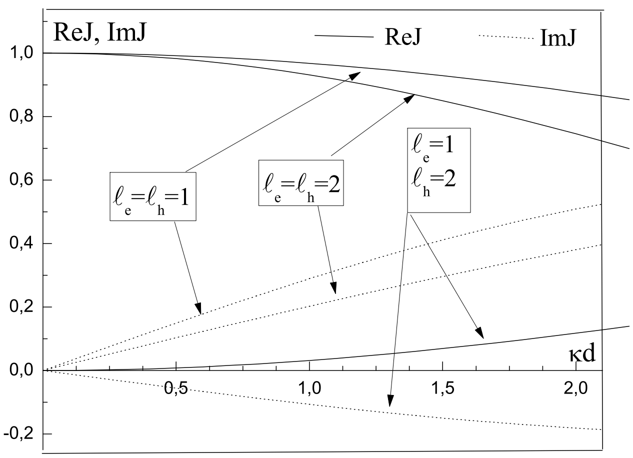

FIG. 2: Dependence of radiative damping $\tilde{\gamma}_r$ (continuous lines) and radiative shift Δ of EHP energy (dashed lines) on a QW width d. On an axis of ordinates $ReJ = \tilde{\gamma}_r/\gamma_r$ and $ImJ = 2\Delta/\gamma_r$ are shown, J is calculated in (65). $\ell_{e(h)}$ is the size quantization quantum number of electron (hole), κ is the module of a light wave vector.

1/2. The factor Q_{0p} depends strongly from the value λ deviation of the cyclotron frequency from the resonant value Ω_e (see Fig.3 in [21]).

In expressions (26) and (27) the approximation $\omega_\rho \simeq \omega_g$ is used, what corresponds to the effective mass method.

At last, the values γ_ρ are non-radiative broadenings of excitations with an index ρ. In [21] their estimations from below for terms $p = a$ and $p = b$ are given.

III. CALCULATION OF ELECTRIC FIELDS IN CASE OF TWO EXITED ENERGY LEVELS

Let us limit the sum in (15) on numbers of exited levels by two terms: $i = 1, 2$. It is possible, if two levels 1 and 2 are located closely to each other and other levels are far from them- on the distance $\Delta\omega$. Thus,

$$\gamma_{1(2)} << |\Delta\omega|, \quad \gamma_{r1(2)} << |\Delta\omega|. \tag{29}$$

The level 1 is characterized by indexes π_1, χ_1, level 2 - by indexes π_2, χ_2. Having entered new designations, we rewrite (15) for a case of two levels as:

$$\mathcal{E}(z,\omega) = \mathcal{E}_0(z,\omega) + \Delta\mathcal{E}(z,\omega), \tag{30}$$

$$\begin{aligned}\Delta\mathcal{E}(z,\omega) \;&=\; -\frac{i}{2}\{\gamma_{r1}M_1(\omega)F_1(z)L_1(\omega) \\ &+\; \gamma_{r2}M_2(\omega)F_2(z)L_2(\omega)\},\end{aligned} \tag{31}$$

where $\Delta\mathcal{E}(z,\omega)$ is the Fourier-component of a field, which we name "induced" as against components of an exciting field $\mathcal{E}_0(z,\omega)$,

$$\gamma_{r\,i} = \gamma_{r\pi_i}, \quad \phi_i = \phi_{\chi_i}, \quad \rho \to i,$$

$$M_i(\omega) = \int_{-d/2}^{d/2} dz\phi_i(z)\mathcal{E}(z,\omega), \tag{32}$$

$$\begin{aligned}F_i(z) \;&=\; e^{i\kappa z}\int_{-d/2}^{z} dz' e^{-i\kappa z'}\phi_i(z') \\ &+\; e^{-i\kappa z}\int_{z}^{d/2} dz' e^{i\kappa z'}\phi_i(z'),\end{aligned} \tag{33}$$

$$\begin{aligned}L_i(z) \;&=\; (\omega - \omega_i + i\gamma_i/2)^{-1} \\ &+\; (\omega + \omega_i + i\gamma_i/2)^{-1}, \quad i = 1, 2.\end{aligned} \tag{34}$$

The equation (31) describes electric fields at any z, i.e. to the left of QW at $z \le -d/2$, inside QW at $-d/2 \le z \le d/2$ and to the right of QW at $z \ge d/2$. It follows from (31) and (33) that to the left of QW the component $\Delta\mathcal{E}_{left}(z,\omega)$ is proportional to $\exp(-i\kappa z)$, what corresponds to reflected light; to the right of QW the component $\Delta\mathcal{E}(z,\omega)$ is proportional to $\exp(i\kappa z)$, what corresponds to wave extending from left to right. Inside QW the solution is more complicated.

The equations (30) - (31) may be solved by a method of iterations, if the interaction of a stimulating field with an electronic system is weak. Further we show that it is possible to be limited by a lowest approximation under the condition

$$\gamma_{r1(2)} << \gamma_{1(2)}. \tag{35}$$

But we solve the equations (30) - (31) precisely, calculating factors $M_1(\omega)$ and $M_2(\omega)$. For this purpose we multiply (30) consecutively on $\phi_1(z)$ and $\phi_2(z)$ and integrate on z from $-d/2$ up to $d/2$. We obtain the system of two equations

$$\begin{aligned}a_{11}M_1 + a_{12}M_2 = C_1, \\ a_{21}M_1 + a_{22}M_2 = C_2,\end{aligned} \tag{36}$$

where

$$a_{11} = 1 + \frac{i}{2}\gamma_{r1}L_1J_{11},$$

$$a_{12} = \frac{i}{2}\gamma_{r2}L_2J_{12},$$

$$a_{21} = \frac{i}{2}\gamma_{r1}L_1J_{21},$$

$$a_{22} = 1 + \frac{i}{2}\gamma_{r2}L_2J_{22},$$

$$C_i = \int_{-d/2}^{d/2} dz\, \mathcal{E}_0(z,\omega)\, \phi_i(z), \qquad (37)$$

$$J_{i\,i'} = \int_{-d/2}^{d/2} dz \phi_i(z) F_{i'} e. \qquad (38)$$

Solving the equation system (36) we obtain

$$M_1 = \frac{C_1 a_{22} - C_2 a_{12}}{a_{11}a_{22} - a_{12}a_{21}}, \quad M_2 = \frac{C_2 a_{11} - C_1 a_{21}}{a_{11}a_{22} - a_{12}a_{21}}. \qquad (39)$$

Let us note the important properties of factors C_i and $a_{i\,i'}$. Having substituted (12) in (37), we obtain

$$C_i = 2\pi E_0 \mathcal{D}_0(\omega) \mathcal{R}^*_{\chi_i}(\kappa), \qquad (40)$$

where

$$R_i(\kappa) = R_{\chi_i}(\kappa) = \int_{-d/2}^{d/2} dz \;\; exp(-i\kappa z)\; \phi_i(z).$$

Having substituted (33) in (38), we have

$$\begin{aligned} J_{i\,i'} \;&=\; \int_{-d/2}^{d/2} dz \phi_{\chi_i}(z) \left\{ e^{i\kappa z} \int_{-d/2}^{z} dz' e^{-i\kappa z} \phi_{\chi_{i'}}(z') \right. \\ &+\; \left. e^{-i\kappa z} \int_{z}^{d/2} dz' e^{i\kappa z'} \phi_{\chi_{i'}}(z') \right\}, \end{aligned} \qquad (41)$$

what is easy to transform to

$$\begin{aligned} J_{i\,i'} \;&=\; \int_{-d/2}^{d/2} dz e^{i\kappa z} \left\{ \phi_{\chi_i(z)} \int_{-d/2}^{z} dz' e^{-i\kappa z} \phi_{\chi_{i'}}(z') \right. \\ &+\; \left. \phi_{\chi_{i'}}(z) \int_{-d/2}^{z} dz' e^{-i\kappa z'} \phi_{\chi_i}(z') \right\}. \end{aligned} \qquad (42)$$

It follows from (41) and (42)

$$J_{i\,i'} = J_{i'\,i},$$

$$ReJ_{i\,i'} \;=\; \tfrac{1}{2}\{\mathcal{R}^*_{\chi_i}(\kappa)\mathcal{R}_{\chi_{i'}}(\kappa) + \mathcal{R}_{\chi_i}(\kappa)\mathcal{R}^*_{\chi_{i'}}(\kappa)\}, \qquad (43)$$

in particular,

$$ReJ_{i\,i} = |\mathcal{R}_{\chi_i}(\kappa)|^2. \qquad (44)$$

We enter also a designation

$$q_{i\,i'} = ImJ_{i\,i'}, \quad q_{i\,i'}(\kappa = 0) = 0. \qquad (45)$$

Thus, having substituted (39) in (31) and using the formula

$$\Delta\mathbf{E}(z,t) = \frac{\mathbf{e}_l}{2\pi} \int_{-\infty}^{\infty} d\omega e^{-i\omega t} \Delta\mathcal{E}(z,\omega) + c.c., \qquad (46)$$

we basically have solved the task about calculation of induced fields in case of two exited energy levels in a wide QW. Further we consider some special cases.

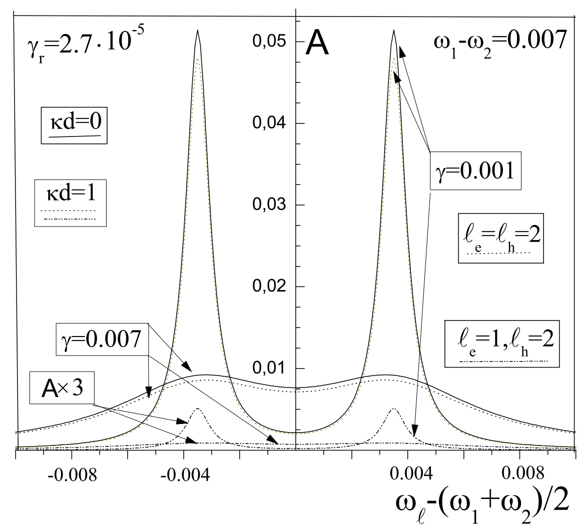

FIG. 3: Dimensionless light reflection $\mathcal{R}$ as function of light frequency ω_ℓ in case of two excitation energy levels in a wide QW under condition $\gamma_r \ll \gamma$. Continuous and dashed lines are the permitted transitions, dot-and-dash lines are the forbidden transitions. $\gamma_r(\gamma)$ is the radiative (non-radiative) broadening of an exited state.

IV. INTERACTION WITH ONE LEVEL

If to put

$$\gamma_{r2} = 0, \qquad (47)$$

it follows from (31) that it remains an interaction of light only with one level. Let us enter designations

$$\begin{aligned} \gamma_{r1} = \gamma_r, \quad \omega_1 = \omega_0, \quad L_1(\omega) = L(\omega), \\ \mathcal{R}_1(\kappa) = \mathcal{R}(\kappa), \quad q_1(\kappa) = q(\kappa) \end{aligned} \qquad (48)$$

and obtain following expressions for fields at the left and to the right of QW:

$$\begin{aligned} \Delta\mathbf{E}_{left}(z,t) \;&=\; -\frac{i}{2}\mathbf{e}_\ell E_0 \int_{-\infty}^{\infty} dt e^{-i\omega t - i\kappa z + i\alpha} \\ &\times\; \frac{\tilde{\gamma}_r(\omega) L(\omega) \mathcal{D}_0(\omega)}{1 + (i\tilde{\gamma}_r(\omega)/2 - \Delta(\omega))L(\omega)} + c.c. \end{aligned} \qquad (49)$$

$$\Delta\mathbf{E}_{right}(z,t) = -\frac{i}{2}\mathbf{e}_\ell E_0 \int_{-\infty}^{\infty} dt e^{-i\omega t + i\kappa z}$$

$$\times \frac{\tilde{\gamma}_r(\omega) L(\omega) \mathcal{D}_0(\omega)}{1 + (i\tilde{\gamma}_r(\omega)/2 - \Delta(\omega))L(\omega)} + c.c., \qquad (49a)$$

where

$$\begin{aligned} \tilde{\gamma}_r(\omega) = \gamma_r |\mathcal{R}(\kappa)|^2, \quad \Delta(\omega) = \gamma_r q(\kappa)/2, \\ e^{i\alpha} = \frac{\mathcal{R}^*(\kappa)}{\mathcal{R}(\kappa)}. \end{aligned} \qquad (50)$$

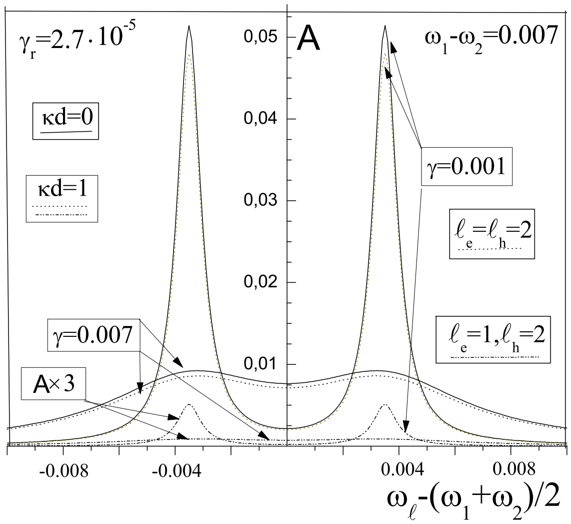

FIG. 4: Dimensionless light absorption $\mathcal{A}$ as function of light frequency ω_ℓ in case of two excitation energy levels in a wide QW under condition $\gamma_r \ll \gamma$. Continuous and dashed lines are the permitted transitions, dot-and-dash lines are the forbidden transitions. $\gamma_r(\gamma)$ is the radiative (non-radiative) broadening of an exited state.

If to use (9) and the function (10), it is possible to show the following. If the size quantization quantum numbers ℓ_e and ℓ_h are of identical parity, $\mathcal{R}^*(\kappa) = \mathcal{R}(\kappa)$ and $e^{i\alpha} = 1$, and if they are different, $\mathcal{R}^*(\kappa) = -\mathcal{R}(\kappa)$ and $e^{i\alpha} = -1$.

If to reject in expression

$$L(\omega) = (\omega - \omega_0 + i\gamma/2)^{-1} + (\omega + \omega_0 + i\gamma/2)^{-1}$$

the non-resonant term $(\omega + \omega_0 + i\gamma/2)^{-1}$, then from (49) and (49a) we obtain

$$\Delta\mathbf{E}_{left}(z,t) = -\frac{i}{2}\mathbf{e}_\ell E_0 \int_{-\infty}^{\infty} d\omega e^{-i\omega t - i\kappa z + i\alpha} \times \frac{\tilde{\gamma}_r(\omega)\mathcal{D}_0(\omega)}{\omega - (\omega_0 + \Delta) + i(\tilde{\gamma}_r + \gamma)/2}, \quad (51)$$

$$\Delta\mathbf{E}_{right}(z,t) = -\frac{i}{2}\mathbf{e}_\ell E_0 \int_{-\infty}^{\infty} d\omega e^{-i\omega t + i\kappa z} \times \frac{\tilde{\gamma}_r(\omega)\mathcal{D}_0(\omega)}{\omega - (\omega_0 + \Delta) + i(\tilde{\gamma}_r + \gamma)/2}. \quad (52)$$

Comparing the first equality from (50) to the expression (24), we find that $\tilde{\gamma}_r(\omega)$ is the radiative broadening of the exited state with energy ω. The value $\Delta(\omega)$ means a certain shift of the exited level in QW caused by interaction with light.

If in [19,20] to put $\nu_1 = \nu$ (where $\nu_1(\nu)$ is the refraction index of a QW (barrier) substance), we obtain the results (51) and (52) .[35]

In case of narrow QWs ($\kappa d << 1$) we obtain

$$\tilde{\gamma}_r \simeq \gamma_r \int_{-d/2}^{d/2} dz\phi(z), \quad exp(i\alpha) \simeq 1, \quad \Delta(\omega) = 0, \quad (53)$$

and for free electrons and holes (when (9) is carried out) $\tilde{\gamma}_r = \gamma_r \delta_{\ell_e,\ell_h}$, the expressions (51) and (52) pass in used, for example, in [16].

Let us emphasize that in case of wide QWs there is a dependence of induced fields on a QW width d (through values $\tilde{\gamma}_r(\omega)$ and $\Delta(\omega)$), and also an opportunity of interaction of light with EHPs, which have $\ell_e \neq \ell_h$, appears.

V. TWO ENERGY LEVELS IN A NARROW QW

In narrow QWs ($\kappa d << 1$) under the condition (9) we obtain from (37)

$$C_i = 2\pi E_0 \mathcal{D}_0 \delta_{\ell_{e\,i},\ell_{h,\,i}}, \quad (54)$$

and it follows from (43)

$$J_{i\,i} = \delta_{\ell_{e\,i},\ell_{h,\,i}}, \quad J_{12} = J_{21} = \delta_{\ell_{e1},\ell_{h1}}\delta_{\ell_{e2},\ell_{h2}}, \quad (55)$$

i.e. light interacts with two levels, which size quantization quantum numbers of electron and hole are identical, i.e.

$$\ell_{e1} = \ell_{h1}, \quad \ell_{e2} = \ell_{h2}.$$

We obtain also

$$M_1 = M_2 = \frac{2\pi E_0 \mathcal{D}_0(\omega)}{1 + (i/2)[\gamma_{r1}L_1(\omega) + \gamma_{r2}L_2(\omega)]}. \quad (56)$$

Having substituted (56) in (31), we find

$$\Delta\mathcal{E}(z,\omega) = -i\pi E_0 \mathcal{D}_0(\omega) \times \frac{\gamma_{r1}L_1(\omega)F_1(z) + \gamma_{r2}L_2(\omega)F_2(z)}{1 + (i/2)[\gamma_{r1}L_1(\omega) + \gamma_{r2}L_2(\omega)]}. \quad (57)$$

Having substituted (57) in (46), we determine the induced electric fields at the left and to the right of QW:

$$\Delta\mathbf{E}_{left}(z,t) = e_l E_0 \int_{-\infty}^{\infty} d\omega e^{-i\omega t - i\kappa z}\mathcal{D}(\omega) + c.c., \quad (58)$$

$$\Delta\mathbf{E}_{right}(z,t) = e_l E_0 \int_{-\infty}^{\infty} d\omega e^{-i\omega t + i\kappa z}\mathcal{D}(\omega) + c.c., \quad (58a)$$

where

$$\mathcal{D}(\omega) = -\frac{4\pi\chi(\omega)\mathcal{D}_0(\omega)}{1 + 4\pi\chi(\omega)}, \quad (59)$$

$$\chi(\omega) = \frac{i}{8\pi}[\gamma_{r1}L_1(\omega) + \gamma_{r2}L_2(\omega)]. \quad (60)$$

For an electric field inside a QW we obtain the result

$$\begin{aligned}\Delta\mathbf{E}_{QW}(z,t) &= -\frac{i}{2}\mathbf{e}_l E_0 \int_{-\infty}^{\infty} d\omega e^{-i\omega t}\mathcal{D}_0(\omega)\\ &\times \{1+(i/2)[\gamma_{r1}L_1(\omega)+\gamma_{r2}L_2(\omega)]\}^{-1}\\ &\times \left\{\gamma_{r1}L_1(\omega)\left[e^{i\kappa z}\int_{-d/2}^{z} dz'\phi_1(z')\right.\right.\\ &+ \left. e^{-i\kappa z}\int_{z}^{d/2} dz'\phi_1(z')\right]\\ &+ \gamma_{r2}L_2(\omega)\left[e^{i\kappa z}\int_{-d/2}^{z} dz'\phi_2(z')\right.\\ &+ \left.\left. e^{-i\kappa z}\int_{z}^{d/2} dz'\phi_2(z')\right]\right\}. \qquad (61)\end{aligned}$$

Obviously, formulas (58), (58a) and (61) may be generalized on any number of exited levels in narrow QWs (see, for example, [18]). Expressions (58), (58a) were used in [17,21].

VI. TWO LEVELS IN A WIDE QW. ELECTRIC FIELDS.

Let us consider a special case, when functions $\phi_{\chi_i}(z)$ for two levels coincide, i.e.

$$\phi_{\chi_1}(z) = \phi_{\chi_2}(z) = \phi(z). \qquad (62)$$

If (9) is carried out, (62) is right, in particular, at

$$\ell_{e1} = \ell_{e2} = l_e, \quad \ell_{h1} = \ell_{h2} = \ell_h. \qquad (63)$$

Let us admit that only γ_{r1} and γ_{r2}, energy $\hbar\omega_1$ and $\hbar\omega_2$ (see [21]) and quantum numbers ℓ_e and ℓ_h may differ each from other. Two close located levels of a system consisting of an usual magnetopolaron and a hole concern to such cases.

With the help of (31) and (39) we obtain

$$\begin{aligned}\Delta\mathcal{E}(z,\omega) &= -i\pi E_0\mathcal{D}_0(\omega)\\ &\times \frac{[\gamma_{r1}L_1(\omega)+\gamma_{r2}L_2(\omega)]F(z)}{1+\frac{i}{2}[\gamma_{r1}L_1(\omega)+\gamma_{r2}L_2(\omega)]J(\kappa)}, \qquad (64)\end{aligned}$$

$L_i(\omega)$ is determined in (34), $F(z)$ is determined in (33),

$$\begin{aligned}J(\kappa) &= \int_{-d/2}^{d/2} dz\phi(z)\left\{e^{i\kappa z}\int_{-d/2}^{z} dz' e^{-i\kappa z'}\phi(z')\right.\\ &+ \left. e^{-i\kappa z}\int_{z}^{d/2} dz' e^{i\kappa z'}\phi(z')\right\}, \qquad (65)\end{aligned}$$

$$J(\kappa) = |\mathcal{R}(\kappa)|^2 + iq(\kappa). \qquad (66)$$

For induced electrical fields on the left and to the right of QW with the help (46) and (64) we find:

$$\Delta\mathbf{E}_{left}(z,t) = \mathbf{e}_l E_0 \int_{-\infty}^{\infty} d\omega e^{-i\omega t - i\kappa z + i\alpha}\tilde{\mathcal{D}}(\omega) + c.c., \qquad (67)$$

$$\Delta\mathbf{E}_{right}(z,t) = \mathbf{e}_l E_0 \int_{-\infty}^{\infty} d\omega e^{-i\omega t + i\kappa z}\tilde{\mathcal{D}}(\omega) + c.c., \qquad (67a)$$

$$\tilde{\mathcal{D}}(\omega) = -\frac{i}{2}\mathcal{D}_0(\omega)[\tilde{\gamma}_{r1}(\omega)L_1(\omega)+\tilde{\gamma}_{r2}(\omega)L_2(\omega)]$$

$$\times\{1-\Delta_1(\omega)L_1(\omega)-\Delta_2(\omega)L_2(\omega)$$

$$+(i/2)[\tilde{\gamma}_{r1}(\omega)L_1(\omega)+\tilde{\gamma}_{r2}(\omega)L_2(\omega)]\}^{-1},$$

where

$$\begin{aligned}\tilde{\gamma}_{r1(2)}(\omega) &= \gamma_{r1(2)}|\mathcal{R}(\kappa)|^2,\\ \Delta_{1(2)}(\omega) &= \frac{1}{2}\gamma_{r1(2)}q(\kappa),\\ e^{i\alpha} &= \frac{\mathcal{R}^*(\kappa)}{\mathcal{R}(\kappa)}. \qquad (68)\end{aligned}$$

We reject non-resonant contributions to $L_1(\omega)$ and $L_2(\omega)$ and obtain

$$\begin{aligned}\tilde{\mathcal{D}}(\omega) &= -iD_0(\omega)\\ &\times \left(\frac{\bar{\gamma}_{r1}/2}{\omega-\Omega_1+iG_1/2}+\frac{\bar{\gamma}_{r2}/2}{\omega-\Omega_2+iG_2/2}\right), \qquad (69)\end{aligned}$$

where $o_{1(2)} = \Omega_{1(2)} - iG_{1(2)}/2$, ($\Omega$ and G are real by definition) satisfy to the equation

$$\begin{aligned}(&\ o-\omega_1+i\gamma_1/2)(o-\omega_2+i\gamma_2/2)\\ - &\ \Delta_1(o-\omega_2+i\gamma_2/2)\\ - &\ \Delta_2(o-\omega_1+i\gamma_1/2)+i(\tilde{\gamma}_{r1}/2)(o-\omega_2+i\gamma_2/2)\\ + &\ i(\tilde{\gamma}_{r2}/2)(o-\omega_1+i\gamma_1/2)=0 \qquad (70)\end{aligned}$$

and are equal

$$\begin{aligned}(\Omega &- iG/2)_{1(2)} = \frac{1}{2}\{\tilde{\omega}_1+\tilde{\omega}_2\\ &\pm \sqrt{(\tilde{\omega}_1-\tilde{\omega}_2)^2+(2\Delta_1-i\tilde{\gamma}_{r1})(2\Delta_2-i\tilde{\gamma}_{r2})}\} \qquad (71)\end{aligned}$$

the sign plus concerns to the subscript 1, and the sign minus - to the subscript 2,

$$\tilde{\omega}_{1(2)} = \omega_{1(2)} + \Delta_{1(2)} - i\Gamma_{1(2)}/2, \qquad (72)$$

$$\Gamma_{1(2)} = \tilde{\gamma}_{r1(2)} + \gamma_{1(2)}. \qquad (73)$$

The expression in the root in (71), generally speaking, is complex. Designations are entered also

$$\bar{\gamma}_{r1} = \tilde{\gamma}_{r1} + \Delta\gamma, \quad \bar{\gamma}_{r2} = \tilde{\gamma}_{r2} - \Delta\gamma, \qquad (74)$$

$$\Delta\gamma = \frac{\tilde{\gamma}_{r1}[\Omega_2 - \omega_2 - i(G_2 - \gamma_2)]}{\Omega_1 - \Omega_2 + i(G_2 - G_1)/2} + \frac{\tilde{\gamma}_{r2}[\Omega_1 - \omega_1 - i(G_1 - \gamma_1)]}{\Omega_1 - \Omega_2 + i(G_2 - G_1)/2}. \quad (75)$$

Let us compare results (67) and (67a) with substitution (69) to the formulas (51) and (52) for a case of one excitation level in a wide QW. It is visible that levels with numbers 1 and 2 influence each other, that results into a renormalization of $\omega_i + \Delta_i$, Γ_i and γ_{ri}: They are being replaced accordingly by Ω_i, G_i and $\bar{\gamma}_{ri}$. In narrow QWs at $\kappa d \ll 1$ $\Delta_1 \cong \Delta_2 \cong 0$, $\exp(i\kappa d) = 1$, and at performance (9), i.e. without the Coulomb forces account,

$$\tilde{\gamma}_{r1} = \gamma_{r1}\delta_{\ell_e,\ell_h}, \qquad \tilde{\gamma}_{r2} = \gamma_{r2}\delta_{\ell_e,\ell_h}, \quad (76)$$

and we obtain results of [21] for two levels in a narrow QW.

VII. TWO ENERGY LEVELS IN A WIDE QW. LIGHT REFLECTION AND ABSORPTION AT MONOCHROMATIC IRRADIATION

Knowing expressions for electric fields on the left and to the right of QW, let us calculate light reflection and absorption. Let us introduce Umov-Pointing vectors $\mathbf{S}_{left(right)}$ on the left (to the right) from QW

$$\mathbf{S}_{left} = \mathbf{S}_0 + \Delta\mathbf{S}_{left}, \quad (77)$$

$$\mathbf{S}_{right} = \frac{c\nu}{4\pi}(\mathbf{E}_0 + \Delta\mathbf{E}_{right})^2\mathbf{e}_z, \quad (78)$$

$$\Delta\mathbf{S}_{left} = -(\Delta\mathbf{E}_{left})^2\frac{c\nu}{4\pi}\mathbf{e}_z, \quad (79)$$

$$\mathbf{S}_0 = \frac{c\nu}{4\pi}E_0^2\mathbf{e}_z. \quad (80)$$

Dimensionless light reflection is defined as

$$\mathcal{R} = \frac{|\Delta\mathbf{S}_{left}|}{|\mathbf{S}_0|}. \quad (81)$$

Light absorption is defined as

$$\mathcal{A} = \frac{|\Delta\mathbf{S}_{left} - \Delta\mathbf{S}_{right}|}{|\mathbf{S}_0|}. \quad (82)$$

Light transmission is equal

$$\mathcal{T} = \frac{|\mathbf{S}_{right}|}{|\mathbf{S}_0|} = 1 - \mathcal{R} - \mathcal{A}. \quad (83)$$

Let us consider a case of monochromatic irradiation of a QW, when (13) is carried out. With the help of (67), (67a), (80). (81) and (82) we find

$$\mathcal{R} = \frac{1}{4Z}\{[\tilde{\gamma}_{r1}(\omega_l - \omega_2) + \tilde{\gamma}_{r2}(\omega_l - \omega_1)]^2 + \frac{1}{4}(\tilde{\gamma}_{r1}\gamma_2 + \tilde{\gamma}_{r2}\gamma_1)^2\}, \quad (84)$$

$$\mathcal{A} = \frac{1}{2Z}\{\tilde{\gamma}_{r1}\gamma_1[(\omega_l - \omega_2)^2 + \gamma_2^2/4] + \tilde{\gamma}_{r2}\gamma_2[(\omega_l - \omega_1)^2 + \gamma_1^2/4] + (\Delta_1\tilde{\gamma}_{r2} - \Delta_2\tilde{\gamma}_{r1}) \times [(\omega_l - \omega_2)\gamma_1 - (\omega_l - \omega_1)\gamma_2]\}, \quad (85)$$

where

$$\tilde{\gamma}_{r1(2)} = \tilde{\gamma}_{r1(2)}(\omega_l) \cong \tilde{\gamma}_{r1(2)}(\omega_g),$$
$$\Delta_{1(2)} = \Delta_{1(2)}(\omega_l) \cong \Delta_{1(2)}(\omega_g),$$
$$Z = [(\omega_l - \Omega_1)^2 + G_1^2/4][(\omega_l - \Omega_2)^2 + G_2^2/4]. \quad (86)$$

Using (71) - (73), we transform (86) to

$$Z = \{(\omega_l - \omega_1)(\omega_l - \omega_2) - \frac{1}{4}(\tilde{\gamma}_{r1}\gamma_2 + \tilde{\gamma}_{r2}\gamma_1 + \gamma_1\gamma_2) - \Delta_1(\omega_l - \omega_2) - \Delta_2(\omega_l - \omega_1)\}^2 + \frac{1}{4}\{(\omega_l - \omega_1)\Gamma_2 + (\omega_l - \omega_2)\Gamma_1 - \Delta_1\gamma_2 - \Delta_2\gamma_1\}^2. \quad (87)$$

Let us emphasize that expressions (84) and (85) for light reflection and absorption, as well as expressions (67) and (67a) for induced electric fields, are right under condition (8), but performance of the condition (9) is not necessarily. The Coulomb forces may be taken into account, therefore functions $\phi_{\chi_i}(z)$ will differ from (9). In narrow QWs at $\kappa d \ll 1$ we obtain

$$\mathcal{R}_\chi = \int_{-\infty}^{\infty} dz\phi_\chi(z), \quad \Delta_\chi \simeq 0. \quad (88)$$

Let us consider some extrem cases.

1. Under condition $\kappa d << 1$, i.e. for narrow QWs, we use (76). Then the expressions (84) and (85) pass in appropriate formulas from [21]. Let us emphasize that in the limit $\kappa d << 1$ for free EHPs only $\ell_e = \ell_h$ is admitted.

2. Having put $\gamma_{r2} = 0$, we exclude interaction of light with the level 2. Then we obtain results for one level

$$\mathcal{R} = \frac{\tilde{\gamma}_{r1}^2}{4[(\omega_l - \omega_1 - \Delta_1)^2 + (\tilde{\gamma}_{r1} + \gamma_1)^2/4]}, \quad (89)$$

$$\mathcal{A} = \frac{\tilde{\gamma}_{r1}\gamma_1}{2[(\omega_l - \omega_1 - \Delta_1)^2 + (\tilde{\gamma}_{r1} + \gamma_1)^2/4]}. \quad (90)$$

3. Let us consider an extreme case

$$\tilde{\gamma}_{r1(2)} << \gamma_{1(2)}, \quad \Delta_{1(2)} << \gamma_{1(2)}, \quad (91)$$

when the perturbation theory on light interaction with an electronic system is applicable. From (84) and (85) we find

$$\mathcal{R} = \frac{(\tilde{\gamma}_{r1}/2)^2}{(\omega_l - \omega_1)^2 + (\gamma_1/2)^2} + \frac{(\tilde{\gamma}_{r2}/2)^2}{(\omega_l - \omega_2)^2 + (\gamma_2/2)^2} + \frac{\tilde{\gamma}_{r1}\tilde{\gamma}_{r2}}{2}\frac{(\omega_l - \omega_1)(\omega_l - \omega_2) + \gamma_1\gamma_2/4}{[(\omega_l - \omega_1)^2 + (\gamma_1/2)^2][(\omega_l - \omega_2)^2 + (\gamma_2/2)^2]}, \quad (92)$$

$$\mathcal{A} \simeq \frac{\tilde{\gamma}_{r1}\gamma_1/2}{(\omega_l-\omega_1)^2+(\gamma_1/2)^2} + \frac{\tilde{\gamma}_{r2}\gamma_2/2}{(\omega_l-\omega_2)^2+(\gamma_2/2)^2}. \quad (93)$$

Since

$$\begin{aligned} \tilde{\gamma}_{r1(2)} &= \gamma_{r1(2)}|\mathcal{R}(\kappa_l)|^2, \\ \mathcal{R}(\kappa_l) &= \int_{-\infty}^{\infty} dz e^{-i\kappa_l z}\phi(z), \quad \kappa_l = \frac{\omega_l \nu}{c}, \end{aligned} \quad (94)$$

and reflection (92) is square on values $\tilde{\gamma}_r$, in the RHS of (92) the factor $|\mathcal{R}(\kappa_l)|^4$ may be taken out of brackets. Similarly in (93) it is possible to bear out brackets $|\mathcal{R}(\kappa_l)|^2$. In brackets $\tilde{\gamma}_{r1(2)}$ should be replaced by $\gamma_{r1(2)}$. Thus, dependence of $\mathcal{R}$ and $\mathcal{A}$ on QW width is determined by factors $|\mathcal{R}(\kappa_l)|^4$ and $|\mathcal{R}(\kappa_l)|^2$, respectively.

For free EHPs in a case $\kappa_l d \geq 1$ there appears an opportunity of light interaction with EHPs for which $\ell_e \neq \ell_h$.

According to (93) light absorption $\mathcal{A}$ is the sum of contributions from levels 1 and 2. Each of these contributions may be obtained from (90) at $\tilde{\gamma}_r << \gamma, \ \ \Delta << \gamma$.

According to (92) light reflection $\mathcal{R}$ is square on $\tilde{\gamma}_{r1}$ and $\tilde{\gamma}_{r2}$ and consequently, except of contributions of separate levels, contains an interference contribution.

4. The following limiting case is opposite to previous and is determined by conditions

$$\gamma_{r1(2)} >> \gamma_{1(2)}, \quad \Delta_{1(2)} >> \gamma_{1(2)}. \quad (95)$$

Let us assume in the RHS of (84) and (85)

$$\gamma_1 = \gamma_2 = 0, \quad (96)$$

then $\mathcal{A} = 0$,

$$\begin{aligned} \mathcal{R} &= [(\tilde{\gamma}_{r1}+\tilde{\gamma}_{r2})/2]^2(\omega_l-\Omega_0)^2 \\ &\times \{[(\omega_l-\omega_1)(\omega_l-\omega_2) \\ &- \Delta_1(\omega_l-\omega_2) - \Delta_2(\omega_l-\omega_1)]^2 \\ &+ [(\tilde{\gamma}_{r1}+\tilde{\gamma}_{r2})/2]^2(\omega_l-\Omega_0)^2\}^{-1}, \end{aligned} \quad (97)$$

where

$$\Omega_0 = \frac{\omega_1\tilde{\gamma}_{r2}+\omega_2\tilde{\gamma}_{r1}}{\tilde{\gamma}_{r1}+\tilde{\gamma}_{r2}}. \quad (98)$$

Having determined roots of the equation

$$(\omega_l-\omega_1)(\omega_l-\omega_2) - \Delta_1(\omega_l-\omega_2 1)$$

$$-\Delta_2(\omega_l-\omega_1) = 0,$$

let us transform (97) to

$$\begin{aligned} \mathcal{R} &= [(\tilde{\gamma}_{r1}+\tilde{\gamma}_{r2})/2]^2(\omega_l-\Omega_0)^2 \\ &\times \{(\omega_l-\omega_{d1})^2(\omega_l-\omega_{d2})^2 \\ &+ [(\tilde{\gamma}_{r1}+\tilde{\gamma}_{r2})/2]^2(\omega_l-\Omega_0)^2\}^{-1}, \end{aligned} \quad (99)$$

where

$$\begin{aligned} \omega_{d1(2)} &= \frac{1}{2}\{\omega_1+\Delta_1+\omega_2+\Delta_2 \\ &\pm \sqrt{(\omega_1+\Delta_1-\omega_2-\Delta_2)^2+4\Delta_1\Delta_2}\}. \end{aligned} \quad (100)$$

It follows from (99) that $\mathcal{R}=0$ at $\omega_\ell = \Omega_0$, i.e. there is a point of total light transmission through a QW. At $\omega_\ell = \omega_{d1}$ or $\omega_\ell = \omega_{d2}$ $\quad \mathcal{R}=1$, i.e. light is completely reflected. Let us compare this result (99) with the result of [21] for narrow QWs ($\kappa d << 1$). We find that at transition to a case of wide QWs values $\tilde{\gamma}_{r1(2)}$ begin to depend on a QW width, the interaction with EHPs for which $\ell_e \neq \ell_h$ is admitted and points of total reflection are displaced (at $\kappa d << 1$ these points are $\omega_\ell = \omega_1$ and $\omega_\ell = \omega_2$).

But existence of one point of total transmission and two points of total reflection in case of wide QWs are preserved (see Fig.7 below).

5. Following extreme case is the smallness of broadenings and energy shifts in comparison to distance $\omega_1 - \omega_2$ between energy levels, i.e.

$$\begin{aligned} \tilde{\gamma}_{r1(2)} << \omega_1-\omega_2, \quad \Delta_{1(2)} << \omega_1-\omega_2, \\ \gamma_{1(2)} << \omega_1-\omega_2, \end{aligned} \quad (101)$$

thus, a ratio between non-radiative broadening $\gamma_{1(2)}$ and values $\tilde{\gamma}_{r1(2)}$ and $\Delta_{1(2)}$ may be anyone.

Let frequency ω_ℓ is close to a resonance with a level 1, i.e. in addition to (101) the conditions are carried out

$$\begin{aligned} \omega_\ell-\omega_1 << \omega_\ell-\omega_2, \quad \Gamma_{1(2)} << \omega_\ell-\omega_2, \\ \Delta_{1(2)} << \omega_\ell-\omega_2. \end{aligned} \quad (102)$$

Then from (84) and (85) we obtain results (89) and (90), i.e. the second level poorly influences $\mathcal{R}$ and $\mathcal{A}$.

6. At last, we consider a case of merging levels, when

$$\begin{aligned} \omega_\ell = \omega_2 = \omega_0, \quad \tilde{\gamma}_{r1} = \tilde{\gamma}_{r2} = \tilde{\gamma}_r, \\ \Delta_1 = \Delta_2 = \Delta, \quad \gamma_1 = \gamma_2 = \gamma. \end{aligned} \quad (103)$$

From (71) we obtain

$$\begin{aligned} \Omega_1 = \omega_0 + 2\Delta, \quad \Omega_2 = \omega_0, \\ G_1 = 2\tilde{\gamma}_r + \gamma, \quad G_2 = \gamma, \end{aligned} \quad (104)$$

Then it follows from (84) - (86)

$$\mathcal{R} = \frac{\tilde{\gamma}_r^2}{(\omega_\ell-\omega_0-2\Delta)^2+(2\tilde{\gamma}_r+\gamma)^2/4}, \quad (105)$$

$$\mathcal{A} = \frac{\tilde{\gamma}_r\gamma}{(\omega_\ell-\omega_0-2\Delta)^2+(2\tilde{\gamma}_r+\gamma)^2/4}. \quad (106)$$

Comparing obtained results with (89) and (90), we find that in case of twice degenerated exited level the formulas for one non-degenerated level with the doubled values of $\tilde{\gamma}_r$ and Δ are right.

VIII. THE CLASSIFICATION OF MAGNETOPOLARONS

In Fig.1 terms of an electron-phonon system in a QW, concerning to a size quantization quantum number ℓ,

are represented by continuous lines. It is supposed that phonons, essential at formation of magnetopolarons (confined or of interface), have one frequency ω_{LO} without the account of dispersion. On an abscissa axis the relation $j^{-1} = \Omega_{e(h)}/\omega_{LO}$ is shown, on an ordinates axis the relation $E/\hbar\omega_{LO}$ is shown, where E is the electron (hole) energy, counted from energy $\varepsilon_\ell^{e(h)}$, appropriate to a size quantization energgy level ℓ.

The polaron states correspond to crossing points of terms. Twofold polarons, appropriate to crossing only of two terms, are designated by black circles. Let us consider some point of terms crossing, to which an integer j corresponds (see (1)). Let n is the Landau quantum number of a level, passing through the given point of terms at $N = 0$ (see Fig.1). Then the conditions should be carried out

$$2j > n \geq j \tag{107}$$

for a twofold polaron existence. It is easy to see that to $j = 1$ one twofold polaron (designated by the letter A) corresponds. Two twofold polarons D and E correspond to $j = 2$ (i.e. $\Omega/\omega_{LO} = 1/2$), three double polarons F, K and L correspond to $j = 3$ (i.e. $\Omega/\omega_{LO} = 1/3$), et cetera.

In Fig.1 polarons, located to the left of $\Omega/\omega_{LO} = 1/3$ are not designated. Above of twofold polarons threefold polarons are located, appropriate to crossing of three terms, more higher fourfold polarons are located and so on. Number of polarons of each sort is equal j at given j . Threefold polarons in bulk crystals are considered for the first time in [28], in QWs - in [29-31].

Let us note that for crossing three and more terms in one point the equidistance of Landau levels is necessary. In order the theory [31] of threefold polarons should be applicable, it is necessary that the amendments to energy, caused by non-parabolicity of bands or by excitonic effect, should be less than splitting of terms. But in case of twofold polarons infringement of parabolisity is not an obstacle, since crossing of two terms all the same exists.

All above mentioned polarons correspond to the integer j. Besides in Fig.1 there are other crossings of lines with quantum number ℓ (continuous lines), designated by empty circles. They correspond to fractional j. As the terms crossed in these points are characterized by $\Delta N \geq 2$, real direct transitions between them with emitting of one phonon are impossible. Let us name such polarons as weak polarons. As the terms are crossed, their splitting is inevitable, but for calculation of splitting we need to take into account transitions between crossing terms through virtual intermediate states or to take into account the small two-phonon contributions in the operator of electron-phonon interaction. In result splittings ΔE_{weak} of terms in case of weak polarons should be much less, than in a case of integer j. The contributions of transitions through intermediate states in ΔE_{weak} are of more high order than $\alpha^{1/2}$ on the Fröhlich dimensionless coupling constant α.

At the account of two or more values of size quantization quantum number ℓ the picture of crossing of terms becomes considerably complicated. Besides usual polarons, appropriate to a level ℓ' (for example polaron A'), occur "combined" polarons for which the electron-phonon interaction connects two electronic levels with different numbers ℓ. The Landau quantum numbers may coincide or be different [32,33]. In Fig.1 for an example three terms concerning to quantum number ℓ' (dot-and-dash lines) and a position of two combined polarons P and Q are shown. In Fig.1 it would be necessary to carry out more dot-and-dash lines and to obtain the greater number of combined polarons. However, it would strongly complicate figure. For an example in Fig.1 the polaron R is designated, which is combined and weak. Interesting feature of combined polarons is that that the appropriate resonant values of magnetic fields depend on distance $\Delta\varepsilon = \varepsilon_{\ell'} - \varepsilon_\ell$ between size quantized levels ℓ and ℓ' and hence on QW depth and width. Really, with the help of Fig.1 it is easily to obtain

$$(\Omega/\omega_{LO})_P = X_P = 1 - (\Delta\varepsilon)/\hbar\omega_{LO},$$
$$(\Omega/\omega_{LO})_Q = X_Q = 1 + (\Delta\varepsilon)/\hbar\omega_{LO}. \tag{108}$$

One more kind of combined polarons [32] is not represented in Fig.1, as it exists only under the condition

$$\Delta\varepsilon = \hbar\omega_{LO}, \tag{109}$$

when, for example, terms $\ell', n = 0, N = 0$ and $\ell, n = 0, N = 1$ coincide at any magnitudes of a magnetic field. For performance of the resonant condition (109) a certain distance between levels ℓ and ℓ' is required, what may be reached only by selection of the QW width and depth.

In order the picture of Fig.1 should be applicable it is necessary that the distances between the neighbour levels $\ell, \ell - 1, \ell + 1$ should be much more, than ΔE of polaron splittings. Since the distances between levels decrease with growth of the QW width d, the restriction from above on d is imposed (Numerical estimations see in [34] (Fig.2 and 3).).

IX. RESULTS OF NUMERICAL CALCULATIONS

It follows from above mentioned classification of magnetopolarons that the technique of calculation of light reflection and absorption stated in section 7 is inapplicable only to combined polarons in a QW, since in case of twofold combined polarons in each of polaron states a or b the states with different numbers ℓ and ℓ' of size quantization "are mixed" and as a result the expression (8) is inapplicable. To all other twofold polarons - usual (black circles) and weakened (empty circles) basic formulas (84) and (85) are applicable.

As excitation 1 a pair consisting from a hole and a state a of polaron appears, as excitation 2 a pair of a hole and state b appears. In narrow QWs characterized by an inequality $\kappa d \ll 1$, light interacts only with those pairs for which the size quantization numbers coincide,

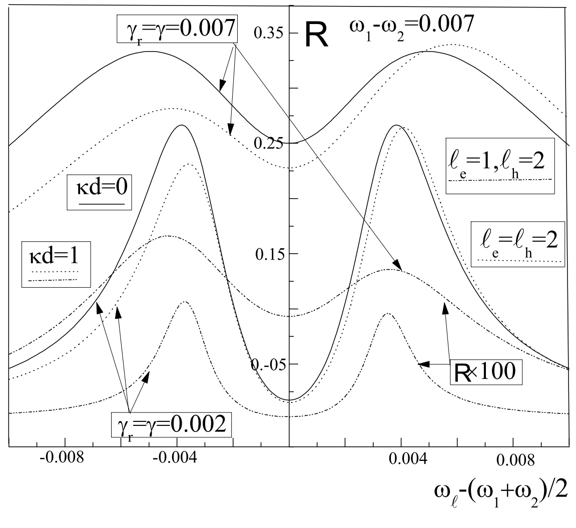

FIG. 5: Same that in Fig.3 under condition $\gamma_r = \gamma$.

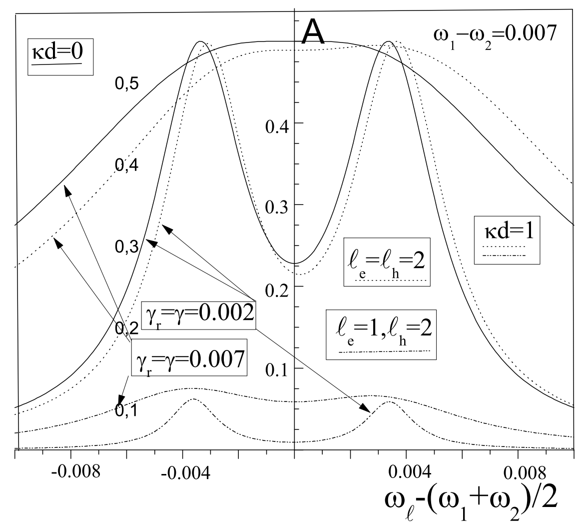

FIG. 6: Same that in Fig.4 under condition $\gamma_r = \gamma$.

i.e. $\ell_e = \ell_h$. Let us name a creation of such EHPs by permitted transitions. For wider QWs (when $\kappa d \geq 1$) forbidden transitions, for which $\ell_e \neq \ell_h$, appear.

Since energies ε_ℓ^e and ε_ℓ^h, determined in (21), depend on numbers ℓ, permitted transitions with indexes $\ell_e = \ell_h = 1$ and $\ell_e = \ell_h = 2$ are separated on frequency on the value

$$D_{eh} = 3\hbar\pi^2/2\mu d^2. \tag{110}$$

For example, for $GaAs$, using parameters [26] $m_e = 0.065\, m_0, m_h = 0.16\, m_0$ and QW width $d = 150\mathring{A}$, we obtain

$$\hbar D_{eh} = 0.11 eV. \tag{111}$$

For wide QWs in an interval between the permitted transitions $\ell_e = \ell_h = 1$ and $\ell_e = \ell_h = 2$ there are two forbidden transitions at $\ell_e = 1, \ell_h = 2$ on distance $3.2 \cdot 10^{-2} eV$ from transition $\ell_e = \ell_h = 1$ and at $\ell_e = 1, \ell_h = 2$ on distance $7.9 \cdot 10^{-2}$ from transition $\ell_e = \ell_h = 1$. To each of transitions at resonant value H_{res} there corresponds a doublet of close located maxima of light reflection and absorption. Comparative intensity of reflection and absorption for cases permitted and forbidden transitions is shown in Fig.3-5.

Let us show still numerical estimations. In [34] the values ΔE of polaron splittings are calculated for some sorts of ordinary double polarons (see Fig.1) and some substances of QWs and barriers.

Dependencies ΔE on a QW width d are constructed.

At d from $150\mathring{A}$ up to $300\mathring{A}$ $\Delta E \approx 5 \div 7 \cdot 10^{-3} eV$ for $\ell_e = \ell_h = 1$ and $\ell_e = \ell_h = 2$. $\hbar\gamma_r$ (according to (26) with use parameters of $GaAs$ from [26]) is equal

$$\hbar\gamma_r \simeq 5.35 \cdot 10^{-5} (H/H_{res}) eV \tag{112}$$

(see also [21]). At $H = H_{res}$ $\quad \gamma_{ra} = \gamma_{rb} = \gamma_r/2$. In Fig.2 the dependence of $\tilde{\gamma}_r$ and radiative energy shifts Δ on QW width d for two permitted transitions $\ell_e = \ell_h = 1$ and $\ell_e = \ell_h = 2$ and two forbidden transitions $\ell = 1, \ell = 2$ and $\ell = 2, \ell = 1$ is represented; results for two forbidden transitions precisely coincide, since according to (9) and (10) the function $\phi_\chi(z)$ does not vary at replacement of an index ℓ_e by ℓ_h and vice verse.[36] At $H = H_{res}$ the non-radiative damping $\gamma_a = \gamma_b$, but their values are unknown, in [21] an attempt is made only to estimate these values from below.

Fig.3 corresponds approximately to above mentioned numerical estimations ΔE and $\gamma_{r1} = \gamma_{r2}$ at $H = H_{res}$.[37] Two arbitrary values $\gamma_1 = \gamma_2 = \gamma$ are used: $\gamma = 0.001$ and $\gamma = 0.007$, both of them exceed $\gamma_{r1} = \gamma_{r2} = 2.7 \cdot 10^{-5}$. So Fig.3 corresponds to a case $\tilde{\gamma}_{r1(2)} \ll \gamma_{1(2)}$. It is visible that $\mathcal{R} \ll 1, \mathcal{A} \ll 1$ and $\mathcal{R} \ll \mathcal{A}$ for permitted and for forbidden transitions.

Comparing the used polaron splittings ΔE_{res} and γ_r for Fig.3 we find that γ_r is two orders less than ΔE_{res}. But it is right only for usual twofold polarons (black circles in Fig.1). In case of weak polarons (the empty circles) polaron splittings are much smaller. Figs.5, 6 and 7, 8 may concern to weak polarons, when ΔE_{res} and γ_r may be comparable. In Figs.5, 6 dependencies $\mathcal{R}(\omega_\ell)$ and $\mathcal{A}(\omega_\ell)$ are represented in a case $\gamma_{r1} = \gamma_{r2} = \gamma_1 = \gamma_2$. For permitted transition at equality of radiative and non-radiative damping $\mathcal{R}$ and $\mathcal{A}$ reach the greatest magnitudes comparable among themselves and comparable with unit, what is visible in Figs.5, 6 . As to forbidden transitions, for them the condition $\tilde{\gamma}_{r1(2)} \ll \gamma_{1(2)}$ is satisfied and appropriate $\mathcal{R}$ and $\mathcal{A}$ are small.

At last, the Figs.7, 8 correspond to an inequality $\gamma_r \gg \gamma$, for which the most interesting results are obtained. In

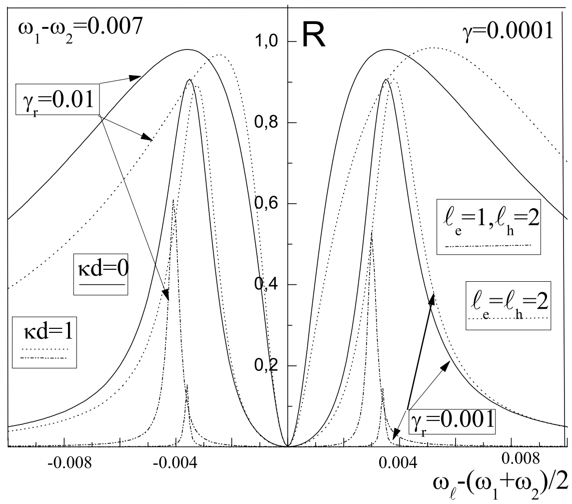

FIG. 7: Same that in Fig.3 under condition $\gamma_r \gg \gamma$.

both cases $\gamma_{r1} = \gamma_{r2} = 0.002$ and 0.01 for a permitted transition on Figs.7, 8 we obtain results appropriate to (99), i.e. equal to zero reflection in a point $\omega_\ell = (\omega_1 + \omega_2)/2$ and total reflection ($\mathcal{R} = 1$) in points $\omega_\ell = \omega_{d1}$ and $\omega_\ell = \omega_{d2}$, which position depends on a QW width.

To the forbidden transition in Figs.7, 8 at $\kappa d = 1$ large $\mathcal{R}$ correspond in maxima. It occurs because $\tilde{\gamma}_{1(2)} \ll \gamma_{r1(2)}$ and $\gamma \simeq 10^{-4}$ are comparable. But peaks are very narrow, since $\tilde{\gamma}_{1(2)} \ll \omega_1 - \omega_2$ and $\gamma_{1(2)} \ll \omega_1 - \omega_2$. In Fig.8, where dependencies $\mathcal{A}(\omega_\ell)$ are represented, to forbidden transition at $\kappa d = 1$ the same high and narrow peaks correspond for the same reason.

X. CONCLUSION

The dimensionless reflection $\mathcal{R}$ and absorption $\mathcal{A}$ are calculated at normal incidence of monochromatic light on a QW surface, which width is comparable to length of a light wave. It is considered a resonance of stimulating radiation with two close located energy levels in a QW, which are two terms of a system consisting from a magnetopolaron and hole. Results are compared with what were obtained earlier in [21] for narrow QWs (which width is much less than a light waves length).

Firstly, it is shown that if a light wave length is comparable to a QW width, height and form of peaks of reflection $\mathcal{R}$ and absorption $\mathcal{A}$ begin to depend on a QW width.

Secondly, it is established, that in case of wide QWs appears an interaction of light with excitations, which are characterized by various size quantization numbers of electrons and holes ($\ell_e \neq \ell_h$). Dependence of $\mathcal{R}$ and $\mathcal{A}$ on a QW width d is various for cases $\ell_e = \ell_h$ and $\ell_e \neq \ell_h$.

Thirdly, it is shown that in case of wide QWs an

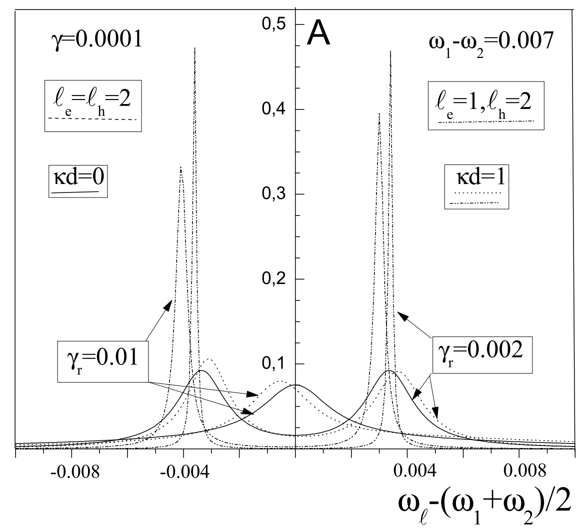

FIG. 8: Same that in Fig.4 under condition $\gamma_r \gg \gamma$.

original behaviour of reflection $\mathcal{R}$ on stimulating light frequency ω_ℓ is preserved, if non-radiative broadenings of excitations are much less than radiative broadenings. There exist two points $\omega_\ell = \omega_{d1}$ and $\omega_\ell = \omega_{d2}$ of total reflection ($\mathcal{R} = 1$) and one point $\omega_\ell = \Omega_0$ between them of total transmission ($\mathcal{R} = 0$). However, the position of points ω_{d1} and ω_{d2} depends on a QW width.

All sequence of processes of absorption and reradiation of light quanta is taken into account, what means an exit outside the perturbation theory on a coupling constant of light and electrons.

It is shown that the perturbation theory is unsuitable, when radiative and non-radiative broadenings are comparable.

XI. ACKNOWLEDGEMENTS

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[35] In [19] in formulas (47) and (48) contain misprints. Instead of γ_r it follows $\tilde{\gamma}_r exp(-i\kappa d/2)$.
[36] It is right only in approximation of indefinitely deep QWs, which is used at construction of figures.
[37] All parameters and frequencies in Fig.3-5 are given in arbitrary units, because the expressions (84) and (85) contain only ratios of these values.